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NONLINEAR OPTIMIZATION PROGRAM PACKAGE

An integrated program package is created to solve nonlinear optimization problems either under or without regional constraints on the control parameters. Four iterative algorithms are included to provide the solution of all the characteristic types of optimization problems. New rational ideas are submitted to improve the already used optimization methods and thus enhance the field of application, reliability and the accuracy of the solution to be achieved.

INTRODUCTION

After the invention of computers optimization methods developed quickly and became widespread in technological practice in the economical and social areas. The availability of a powerful methodology for constructing mathematical models gave the possibility to research and find the optimal working modes of industrial sites not only in real-time conditions but also through "numerical experiments" on a computer [1].

The optimization procedures became an important factor when selecting a rational flowsheet and calculating mass and heat balance and recently for digital simulation of different processes in the field of mineral processing and metalurgy [2,11-15].

Taking into account the complexity of the investigated phenomena and the fact that the optimization problems of chemical and technological mineral processing, and metalurgical sites are described mainly with the help of nonlinear objective functions, the authors directed their efforts to the development of an integrated program system, providing the solutions to a wide class of nonlinear optimization problems, such as:

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finding the optimum of Q(X)

for $x_{i\,min} \le x_i \le x_{i\,max}$, i = 1,2,...,N

and $F_j(X) \ge 0$, j = 0,1,2,...,M

where : Q(X) is the objective function;

x_i is i-th limited control parameter;

$F_j(X)$ is j-th regional constraint.

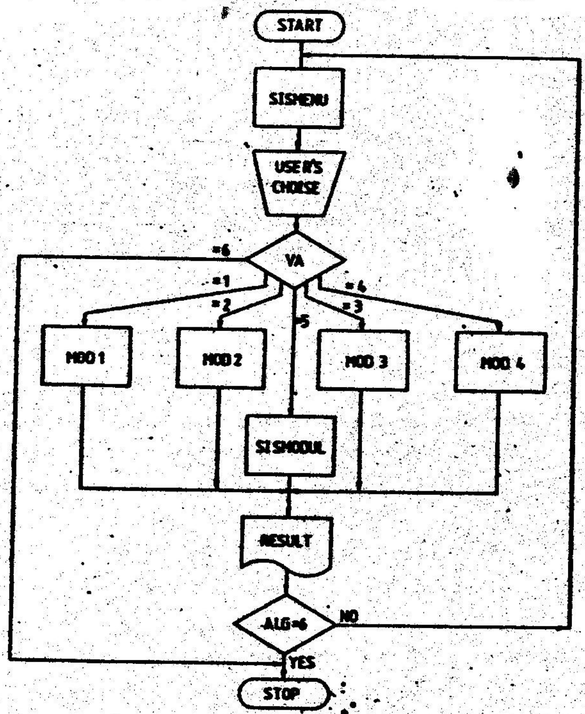

Fig.1 System general flowchart

The system locates a global optimum of the following typical objective functions:

- univarite, multimodal under or without any regional constraints;
- multimodal function of several variables under or without any constraints;
- ridge type function.

Necessary requirements of the objective surface Q(X) are:

- that the function be uninterrupted and defined in the permitted area of control parameters;
- thatin the presumed optimal point, the objective function has defined

partial derivates or no partial derivates [3].

The precise conditions to establish a solution for the problems with constraints are ensured by the conditions of Kuhn-Tucker [16] and Fiacco-McCormick [4].

The maximum number of control parameters x_i and of the constrained functions $F_j(X)$ is 10, i.e. i = 1,2,...,10 and j = 0,1,2,...,10.

GENERAL STRUCTURE OF THE PROGRAM PACKAGE "CONOPTIMA".

The program unit consists of a control module (SYSMENU) and a computing module set called by the control module. The system's general flowchart is shown on Fig.1. Modules MOD1 ÷ MOD4 include particular optimization procedures and everyone of them is designated for a specific class of objective functions. By the user's wish he is provided with the possibility to follow the execution of every module test example. If the super imposed regional constraints are uncompatible, the program system gives a prompt that the problem can't be solved.

The modules are composed according to the general flowchart, illustrated on Fig.2.

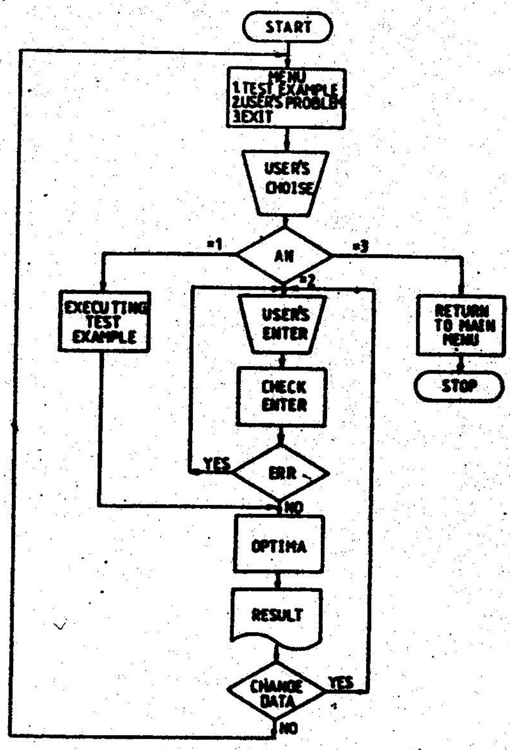

Fig.2. Module composition flowchart

The program package provides to the user the possibility to work with a fixed optimization method (MOD1 v MOD4) or through the algorithm superimposed in the module SYSMOD.

This algorithm includes analysis of the user's problem and submits optimization procedures taking into consideration the problem peculiarities. A possibility is provided for consecutive application of all the proposed methods, comparison of the results and selection of the best one.

MODULE MOD1

This block treats the solution of optimization problems for objective functions with one control parameter. A simplified flowchart of the module is shown on Fig. 3 and the case to find a global maximum is considered.

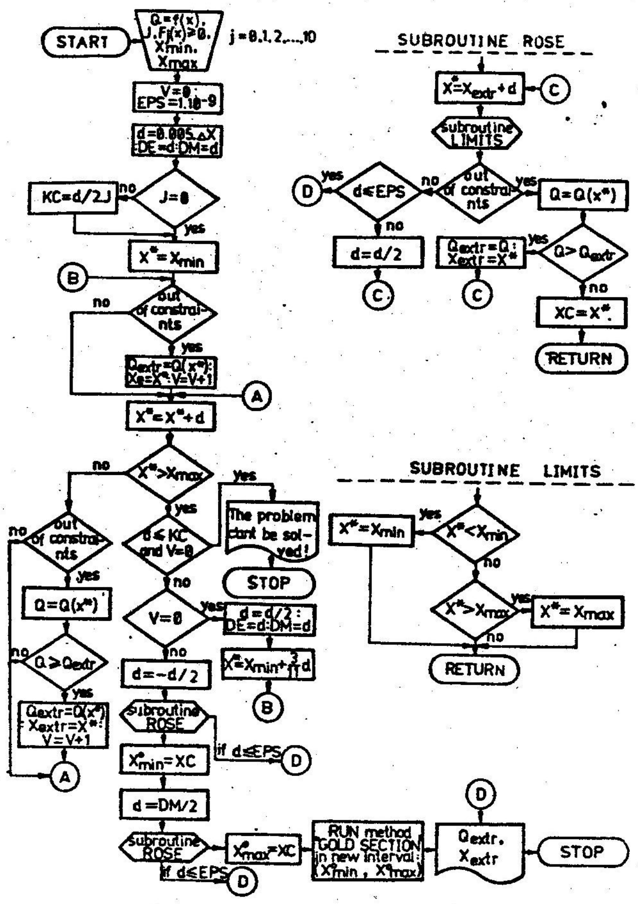

Fig. 3. Simplified flowchart of module MOD 1.

The problem of the numerical solution of that type of task when the regional constraints are missed is considered to be solved. Up to now, several methods of approach have been suggested, including the well known : scanning, dihotommy, golden section, and interpolation methods e.t.c. [5,6,9,17].

Frequently however, additional regional constraints of the type $F_j \geq 0$, $j = 1,2,\dots,M$ are attached to the control parameter's limits. When applying some of those methods it is not always possible to obtain the optimal solution. Because of that fact, one original technique is suggested to define the global optimum of an objective function one control parameter . The algorithm created includes a modified combination of the scanning method with constant step and the golden section method. The foundation of the extreme value is guaranteed with definite accuracy even if it is critically located in relation to the constraints and limits. The algorithm suggested is tested through a great number of examples and has been reliable with a relatively quick convergence and accuracy of the solution.

MODULE MOD2

The module contains an optimization procedure for the location of an optimum for objective functions of the ridge type, Fig. 4. The algorithm is based on the existing well known method of Gelfand & Cetlyn [7]. The main functional unit of this module is the subroutine for numerical finding of the components of the objective surface in a current point, Fig. 5.

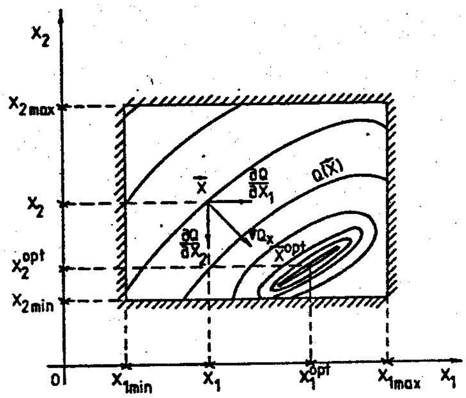

Fig. 4.
Ridge type objective function

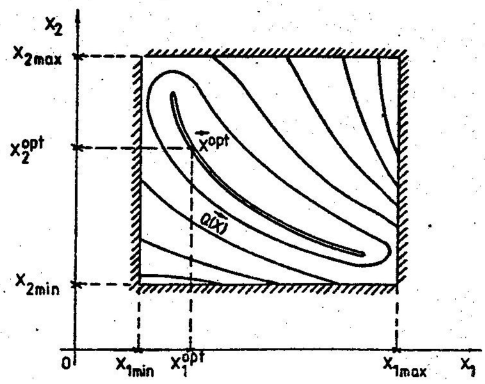

Fig. 5.
Partial derivates and ∇Q vector

Two methods of approach for numerical differentiation are accepted: with singleside and doubleside increments of the argument. In the second case a symmetry is provided regardless of the alocation of the current co-ordinate in relation to the limits. If the value of one of the parameters is on the corresponding limit, the gradient $\vec{\nabla} Q$ is defined from the figures:

- Fig.6a - by means of singleside increment along x_1;
- Fig.6b - $\partial Q / \partial x_1 = 0$, $\partial Q / \partial x_2 = 0$ and the gradient coincides with axis x_2 .

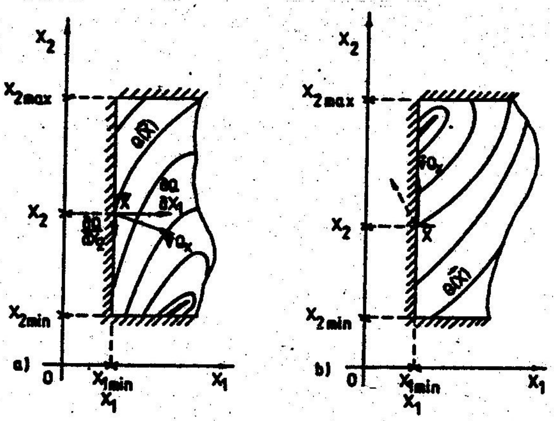

Fig.6. Two cases of computing $\vec{\nabla}Q$ vector.

It should be noticed that a lot of practical problems are described by ridge-type functions because of the very different dimensions of the control parameters. The method of Gelfand & Cetlyn is one effective algorithm for solving such a type of optimization problems, which was confirmed by the authors when it was precisely tested.

MODULE MOD3

In this module an algorithm is realised from the class of the direct methods based on the adaptive random search. The movement towards the optimum is accelerated by means of modification of the algorithm of Rastrigin [8] - random search with back step. The following method of approach is accepted: in case of consecutively falling into the successful step point, the current step to increase proportionally to the already reached number of iterations:

$$h^{(n)}_{oi} = h^{(c)}_{oi} \; (1 + BL / [\; 0{,}5 . (\; 1 + \lg LL \;)])$$

Where: $h^{(c)}_{oi}$ - current increment;

BL -logical variable, having the value:

$$BL = \begin{cases} 1 \text{ for } LL \neq 1 \text{ and } L = 0 \\ 0 \text{ for } LL = 1 \text{ or } L \neq 0 \end{cases}$$

LL - current number of iterations;

$L_{(n)}$ - counter of the consecutive bad step points;

h_{oi} - new value of the increment.

The algorithm quickly convergens and is highly effective for objective functions without the typical peculiarities either under or without any regional constraints.

MODULE MOD4

This module is provided to solve a multioptimal complex (transcendental) objective functions dependent on x_i control parameters ($i = 1,2,...,10$) either under superimposed regional constraints or without any constraints. Essentially, the algorithm is a program combination of the modified complex method of Box [1,18], the method of moving constraints [3,19] and the idea of taking in the limits of every control parameter in relation to the current optimal value, obtained after the successful generation of a complex of points. Unfortunately, this is the slowest algorithm, which gives a solution only after a considerable number

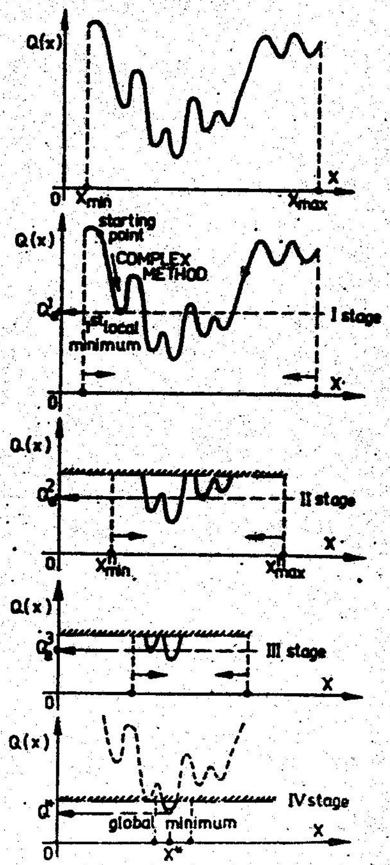

Fig. 7. Stages of locating the extreme for module MOD 4

of iterations (usually over 5000). Its main priority is the universality of the type of the given objective function. The idea of the authors is illustrated in Fig.7 and for simplicity a single parameter objective function is used for which the minimum is search. The algorithm is repeatedly tested with the function of Rosenbrock [20]:

$$Q(X) = x_1 \cdot x_2 \cdot x_3$$

for: $0 \le x_i \le 42$, $i = 1,2,3$

$$F_1(X) = x_1 + 2 \cdot x_2 + 2 \cdot x_3$$

$$F_2(X) = 72 - x_1 - 2 \cdot x_2 - 2 \cdot x_3$$

$$F_j(X) \ge 0 \ , \ j = 1,2.$$

In 87 % of the cases the following result is obtained:

$Q(X) > 3455{,}69$, for theoretical maximum equal to 3456,00.

APPLICATION

Since the mid-nineteen sixties, many researches have been seriously concerned with process models (analytical or empirical) for various ore-dressing problems. Those models can serve to increase process knowledge, assess circuit performance, investigate flowsheets and their modifications, design new flowsheets, perform off/on-line optimization, and compare alternative control strategies.

In the cases used here as examples, the authors are confined only to the most general application of the program package for certain mineral processing purposes.

EXAMPLE 1

Optimizing Mineral Processing Plants and their Operations

In his article [21], J. Hollaway demonstrates the method of residual regression analysis for creating mathematical models of treatment sites in the minerals industry. Using actual data, obtained during the operation of a flotation plant, he studies the effect of a fairly wide range of factors on pyrite recovery. The relationship thus obtained is of the following type:

Predicted Recovery(%) = - 640,46 + 2,636. Feed Grade(%)

+ 17,049. Grind(% - 200 mesh)
- 0,101. $Grind^2$
+ 23,524. Coll.Add. (kg/t)
- 0,037. Throughput(t/day) (1)

The model described (eq. 1) is based on a real understanding of the effects of the operating variables; however, it is not sufficient for the process optimization. It is necessary to use an optimization approach that could result in maximum pyrite recovery at the given factors' limits. The analysis of the model shows a considerable difference in its coefficients, which is characteristic of the ridge type function. That is why the method of Gelfand & Cetlyn (MOD 2) and the modified complex method (MOD 4) are chosen as optimization procedures. The results obtained from the consecutive executions of the two methods are given in table 1.

Table 1.

	OPTIMUM VALUES				
METHOD	Recovery %	Feed Grade g/t	Grind %-200m	Coll. Addition kg/t	Throughput t/day
MOD 2	77,723	5,400	84,401	0,096	480,070
MOD 4	77,746	5,400	84,403	0,096	480,070

Using this approach, in sights can be gained into the plant process and its optimization becomes a matter of scientific deduction. The periodic actualization of the optimum technological regime at a definite interval of time, depending on the complexity of operations and the variety of the ore, will help its rational working.

EXAMPLE 2

Laboratory Studies

The possibility for the separate recovery of ions of heavy non-ferrous metals by sulphidization precipitation and the succeeding stages flotation is shown in [10]. A complete lab factor experiment is made to optimize the conditions of copper precipitation as Cu_2S with NaHS sulphidization agent. The residual concentration of Cu, mg/l in solution is taken as an objective criterion (functional Q), while the independent variables are as follows:

X1 - pH;
X2 - NaHS concentration, mg/l;
X3 - duration of stirring throughout the precipitation, sec;
X4 - stirring rate, min^{-1}

The final model is not drawn in the above cited paper. It is mentioned only that the last two factors (X3 and X4) are insignificant.

By using the standard sophisticated programme (multiple linear regression), the authors obtained the following relation:

$$Q = 6{,}02E\text{-}02 + 0{,}04 \cdot X1 + 1{,}25E\text{-}04 \cdot X1 \cdot X2 - 2{,}50E\text{-}04 \cdot X1 \cdot X3 + 3{,}12E\text{-}06 \cdot X2 \cdot X3 \quad (2)$$

for which the coefficient of multiple correlation is $R \approx 0{,}97$.

The stirring duration effect during precipitation is studied by a random search method (MOD 3). The results are illustrated in table 2.

Table2.

Top Limit of X3, sec	180	190	200	210	220	230	240
RESULTS							
Qmin, mg/l	0,139	0,134	0,130	0,126	0,116	0,107	0,098
X1opt	3,000	3,000	3,000	4.929	5,000	5,000	5,000
X2opt, mg/l	100,0	100,0	100,0	100,0	100,0	100,0	100,0
X3opt, sec	180,0	190,0	200,0	210,0	220,0	230,0	240,0

Definite conclusions can be drawn on the basis of the data obtained;

- the residual concentrarion of Cu^{2+} in the solution decreases with the increase in stirring duration;
- factor X3 has no effect on the optimum NaHS concentration (100mg/l);
- a less acidic medium (0 = 4,5 v 5,0) can be used for optimum characteristics of the process at a prolonged stirring time.

The methodology described gives the possibility for the laboratory experiments to be used as scientific foundations for further investigations in pilot plant and industrial conditions.

EXAMPLE 3

Steady - state Simulation of Mineral Processing Circuits

In recent years, the benefication industry is rapidly changing its scene in terms of high energy costs and ores which have become increasingly difficult to process. This has led to the need for more effective operation of mineral processing systems. As a means of improving process efficiency, computer simulationis being applied in great number to crushing, grinding and flotation circuits [22]. Some of the more important uses of simulation are shown in [23, p.97].

The optimization methods applied in the imitation modelling of a given flowsheet find the following applications:
- for adjustment and material balancing of mineral processing data;
- for development of optimum circuit performance.

When making the mass balances, the collected metallurgical data must be adjusted, since they are often redundant and erroneous due to natural disturbances, sampling errors, unreliable instrument readouts and laboratory analysis inaccuracies.

There are many publications on this topic [11,15,24], and their common point is the minimization of a normalized weighed sum of sqares functional:

$$Q = \sum_i \left[\frac{Wi(Di,g - Di,a)}{Di,j} \right]^2 \qquad i = 1,2,\ldots,N \qquad (3)$$

where: Wi - is the weighed factor (usually equal to an inverse normalized standard derivation estimate);

Di,g - is the i-th given data value;

Di,a - is the i-th adjusted data value;

N - number of data points.

An analogical functional can be made up also when determining the parameters of the model used for process simulation (see ref.[30], p. 933). The authors' opinion is that the optimization procedures MOD 2 + MOD 4 successfully solve the problem for data adjustment. In order toensure maximum efficiency of the operations the application of the program package is necessary also:
- for the optimization of the arrangement of flotation circuits [25];
- for simulating the influence of water addition to a flotation flowsheet [26];
- for investigation on the effect of aeration rate and variations in cell volume [27], and e.t.c.

Detailed information concerning simulation modelling of ore-dressing processes is available in the special literature [23,28,29].

CONCLUSION

The developed program unit is realized on the algorithmic language BASIC. It enriches the program fund for automated solution of nonlinear problems und eror without regional constraints. The necessity of creating it imposed by the fact that the engineer, the technologist and the researcher design models, which are optimization problems in essence. The product provides a possibility for complex research of a concrete mathematical model (function) for different input data (different control parameters, starting point's co-ordinates, starting increments e.t.c.). By means of the selected program algorithm set, the optimization of wide spectrum of technological characteristics is provided when ever an accuracy is as sured one digital more than that assigned by the user. The Program Package, a complete technical descriptions and user's manual are available through TECHNOCOMMERCE, 1113 Sofia, Bulgaria.

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STRESZCZENIE

Danilov L.N, Georgiev C.V., 1988 Problemy optymalizacji nieliniowej w procesach przeróbczych. Fizykochemiczne Problemy Mineralurgii; 20, 273-286

Opracowano program komputerowy dla rozwiązywania nieliniowych problemów optymalizacyjnych z baz lokalnych kontroli parametrów. Przedstawiono cztery algorytmy iteracyjne, które pozwalają rozwiązywać niektóre problemy optymalizycyjne. Przedstawiono również nowe koncepcje dla udoskonalenia istniejących już metod optymalizacyjnych pozwalające poszerzyć zakres ich stosowania oraz niezawodność i dokładność otrzymywanych rozwiązań. Opis matematyczny uzupełniono ogólnymi przykładami z dziedziny przeróbki kopalin.

СОДЕРЖАНИЕ

Л.Н.Данаилов, Ц.В.Георгиев, 1988. Проблемы нелинейной оптимализации в процессах переработки полезных ископаемых. Физикохимические вопросы обогащения, 20; 273-286.

Разработана компьютерная программа для решения нелинейных проблем оптимализации из баз локальных контрольных параметров. Представлены четыре итерративные алгоритмы, позволяющие решать все типы оптимализационных проблем. Представленные новые рациональные идеи для улучшения существующих методов оптимализации, позволяют расширить поле предела их применения, а также надежность и точность получаемых решений.